Tables of Common Transform Pairs

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Engineers and students in communications and mathematics are confronted with transformations such as the z-Transform, the Fourier transform, or the Laplace transform. Often it is quite hard to quickly find the appropriate transform in a book or the Internet, much less to have a comprehensive overview of transformation pairs and corresponding properties.

In this document I compiled a handy collection of the most common transform pairs and properties of the

- **continuous-time frequency Fourier transform** $(2\pi f)$,
- **continuous-time pulsation Fourier transform** (ω),
- **z-Transform**,
- **discrete-time Fourier transform DTFT**, and
- **Laplace transform**.

Please note that, before including a transformation pair in the table, I verified its correctness. Nevertheless, it is still possible that you may find errors or typos. I am very grateful to everyone dropping me a line and pointing out any concerns or typos.

Notation, Conventions, and Useful Formulas

Imaginary unit	$j^2 = -1$
Complex conjugate	$z = a + jb \quad \longleftrightarrow \quad z^* = a - jb$
Real part	$\mathfrak{Re}\{f(t)\} = \frac{1}{2}\left[f(t) + f^*(t)\right]$
Imaginary part	$\mathfrak{Im}\{f(t)\} = \frac{1}{2j}\left[f(t) - f^*(t)\right]$
Dirac delta/Unit impulse	$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$
Heaviside step/Unit step	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$
Sine/Cosine	$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \qquad \cos(x) = \frac{e^{jx} + e^{-jx}}{2}$
Sinc function	$\operatorname{sinc}(x) \equiv \frac{\sin(x)}{x}$ (unnormalized)
Rectangular function	$\operatorname{rect}(\frac{t}{T}) = \begin{cases} 1 & \text{if } \lvert t\rvert \leqslant \frac{T}{2} \\ 0 & \text{if } \lvert t\rvert > \frac{T}{2} \end{cases}$
Triangular function	$\operatorname{triang}\left(\frac{t}{T}\right) = \operatorname{rect}(\frac{t}{T}) * \operatorname{rect}(\frac{t}{T}) = \begin{cases} 1 - \frac{\lvert t\rvert}{T} & \lvert t\rvert \leqslant T \\ 0 & \lvert t\rvert > T \end{cases}$
Convolution	continuous-time: $(f * g)(t) = \int_{-\infty}^{+\infty} f(\tau)\, g^*(t - \tau) d\tau$
	discrete-time: $(u * v)[n] = \sum_{m=-\infty}^{\infty} u[m]\, v^*[n - m]$
Parseval theorem	general statement: $\int_{-\infty}^{+\infty} f(t) g^*(t) dt = \int_{-\infty}^{+\infty} F(f) G^*(f) df$
	continuous-time: $\int_{-\infty}^{+\infty} \lvert f(t)\rvert^2 dt = \int_{-\infty}^{+\infty} \lvert F(f)\rvert^2 df$
	discrete-time: $\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$
Geometric series	$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \qquad \sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$
	in general: $\sum_{k=m}^{n} x^k = \frac{x^m - x^{n+1}}{1-x}$

Table of Continuous-time Frequency Fourier Transform Pairs

	$f(t) = \mathcal{F}^{-1}\{F(f)\} = \int_{-\infty}^{+\infty} f(t)e^{j2\pi ft}df$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-j2\pi ft}dt$	
transform	$f(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)$	
time reversal	$f(-t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(-f)$	frequency reversal
complex conjugation	$f^*(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F^*(-f)$	reversed conjugation
reversed conjugation	$f^*(-t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F^*(f)$	complex conjugation
	$f(t)$ is purely real	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f) = F^*(-f)$	even/symmetry
	$f(t)$ is purely imaginary	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f) = -F^*(-f)$	odd/antisymmetry
even/symmetry	$f(t) = f^*(-t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)$ is purely real	
odd/antisymmetry	$f(t) = -f^*(-t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)$ is purely imaginary	
time shifting	$f(t-t_0)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)e^{-j2\pi ft_0}$	
	$f(t)e^{j2\pi f_0 t}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f-f_0)$	frequency shifting
time scaling	$f(af)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{\lvert a\rvert}F\left(\frac{f}{a}\right)$	
	$\frac{1}{\lvert a\rvert}f\left(\frac{f}{a}\right)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(af)$	frequency scaling
linearity	$af(t) + bg(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$aF(f) + bG(t)$	
time multiplication	$f(t)g(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f) * G(f)$	frequency convolution
frequency convolution	$f(t) * g(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$F(f)G(f)$	frequency multiplication
delta function	$\delta(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	1	
shifted delta function	$\delta(t-t_0)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$e^{-j2\pi ft_0}$	
	1	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\delta(f)$	delta function
	$e^{j2\pi f_0 t}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\delta(f-f_0)$	shifted delta function
two-sided exponential decay	$e^{-a\lvert t\rvert} \quad a > 0$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{2a}{a^2+4\pi^2f^2}$	
	$e^{-\pi t^2}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$e^{-\pi f^2}$	
	$e^{j\pi t^2}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$e^{j\pi\left(\frac{1}{4}-f^2\right)}$	
sine	$\sin(2\pi f_0 t + \phi)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{j}{2}\left[e^{-j\phi}\delta(f+f_0) - e^{j\phi}\delta(f-f_0)\right]$	
cosine	$\cos(2\pi f_0 t + \phi)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{2}\left[e^{-j\phi}\delta(f+f_0) + e^{j\phi}\delta(f-f_0)\right]$	
sine modulation	$f(t)\sin(2\pi f_0 t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{j}{2}\left[F(f+f_0) - F(f-f_0)\right]$	
cosine modulation	$f(t)\cos(2\pi f_0 t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{2}\left[F(f+f_0) + F(f-f_0)\right]$	
squared sine	$\sin^2(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{4}\left[2\delta(f) - \delta\left(f-\frac{1}{\pi}\right) - \delta\left(f+\frac{1}{\pi}\right)\right]$	
squared cosine	$\cos^2(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{4}\left[2\delta(f) + \delta\left(f-\frac{1}{\pi}\right) + \delta\left(f+\frac{1}{\pi}\right)\right]$	
rectangular	$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \lvert t\rvert \leqslant \frac{T}{2} \\ 0 & \lvert t\rvert > \frac{T}{2} \end{cases}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$T\,\mathrm{sinc}\,Tf$	
triangular	$\mathrm{triang}\left(\frac{t}{T}\right) = \begin{cases} 1-\frac{\lvert t\rvert}{T} & \lvert t\rvert \leqslant T \\ 0 & \lvert t\rvert > T \end{cases}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$T\,\mathrm{sinc}^2\,Tf$	
step	$u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{j2\pi f} + \delta(f)$	
signum	$\mathrm{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{j\pi f}$	
sinc	$\mathrm{sinc}(Bt)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{B}\,\mathrm{rect}\left(\frac{f}{B}\right) = \frac{1}{B}1_{\left[-\frac{B}{2},+\frac{B}{2}\right]}(f)$	
squared sinc	$\mathrm{sinc}^2(Bt)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{B}\,\mathrm{triang}\left(\frac{f}{B}\right)$	
n-th time derivative	$\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$(j2\pi f)^n F(f)$	
n-th frequency derivative	$t^n f(t)$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\frac{1}{(-j2\pi)^n}\frac{d^n}{df^n}F(f)$	
	$\frac{1}{1+t^2}$	$\overset{\mathcal{F}}{\Longleftrightarrow}$	$\pi e^{-2\pi\lvert f\rvert}$	

Table of Continuous-time Pulsation Fourier Transform Pairs

	$x(t) = \mathcal{F}_\omega^{-1}\{X(\omega)\} = \int_{-\infty}^{+\infty} x(t)e^{j\omega t}d\omega$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega) = \mathcal{F}_\omega\{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$	
transform	$x(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$	
time reversal	$x(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(-\omega)$	frequency reversal
complex conjugation	$x^*(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X^*(-\omega)$	reversed conjugation
reversed conjugation	$x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X^*(\omega)$	complex conjugation
	$x(t)$ is purely real	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(f) = X^*(-\omega)$	even/symmetry
	$x(t)$ is purely imaginary	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(f) = -X^*(-\omega)$	odd/antisymmetry
even/symmetry	$x(t) = x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$ is purely real	
odd/antisymmetry	$x(t) = -x^*(-t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)$ is purely imaginary	
time shifting	$x(t-t_0)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega)e^{-j\omega t_0}$	
	$x(t)e^{j\omega_0 t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(\omega-\omega_0)$	frequency shifting
time scaling	$x(af)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	
	$\frac{1}{\lvert a\rvert}x\left(\frac{f}{a}\right)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X(a\omega)$	frequency scaling
linearity	$ax_1(t) + bx_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$aX_1(\omega) + bX_2(\omega)$	
time multiplication	$x_1(t)x_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$	frequency convolution
frequency convolution	$x_1(t) * x_2(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$X_1(\omega)X_2(\omega)$	frequency multiplication
delta function	$\delta(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	1	
shifted delta function	$\delta(t-t_0)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$e^{-j\omega t_0}$	
	1	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$2\pi\delta(\omega)$	delta function
	$e^{j\omega_0 t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$2\pi\delta(\omega-\omega_0)$	shifted delta function
two-sided exponential decay	$e^{-a\lvert t\rvert} \quad a > 0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{2a}{a^2+\omega^2}$	
exponential decay	$e^{-at}u(t) \quad \Re\{a\} > 0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{a+j\omega}$	
reversed exponential decay	$e^{-at}u(-t) \quad \Re\{a\} > 0$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{a-j\omega}$	
	$e^{\frac{t^2}{2\sigma^2}}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$	
sine	$\sin(\omega_0 t + \phi)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$j\pi\left[e^{-j\phi}\delta(\omega+\omega_0) - e^{j\phi}\delta(\omega-\omega_0)\right]$	
cosine	$\cos(\omega_0 t + \phi)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi\left[e^{-j\phi}\delta(\omega+\omega_0) + e^{j\phi}\delta(\omega-\omega_0)\right]$	
sine modulation	$x(t)\sin(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$	
cosine modulation	$x(t)\cos(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$	
squared sine	$\sin^2(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi^2\left[2\delta(f) - \delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$	
squared cosine	$\cos^2(\omega_0 t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi^2\left[2\delta(\omega) + \delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$	
rectangular	$\operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \lvert t\rvert \leqslant \frac{T}{2} \\ 0 & \lvert t\rvert > \frac{T}{2} \end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$T\operatorname{sinc}\left(\frac{\omega T}{2}\right)$	
triangular	$\operatorname{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{\lvert t\rvert}{T} & \lvert t\rvert \leqslant T \\ 0 & \lvert t\rvert > T \end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$T\operatorname{sinc}^2\left(\frac{\omega T}{2}\right)$	
step	$u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\pi\delta(f) + \frac{1}{j\omega}$	
signum	$\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{2}{j\omega}$	
sinc	$\operatorname{sinc}(Tt)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{T}\operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T,+\pi T]}(f)$	
squared sinc	$\operatorname{sinc}^2(Tt)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$\frac{1}{T}\operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$	
n-th time derivative	$\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$(j\omega)^n X(\omega)$	
n-th frequency derivative	$t^n f(t)$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$j^n\frac{d^n}{df^n}X(\omega)$	
time inverse	$\frac{1}{t}$	$\overset{\mathcal{F}_\omega}{\Longleftrightarrow}$	$-j\pi\operatorname{sgn}(\omega)$	

Table of z-Transform Pairs

$x[n]=\mathcal{Z}^{-1}\{X(z)\}=\frac{1}{2\pi j}\oint X(z)z^{n-1}dz$		$\overset{\mathcal{Z}}{\longleftrightarrow}$	$X(z)=\mathcal{Z}\{x[n]\}=\sum_{n=-\infty}^{+\infty}x[n]z^{-n}$	*ROC*
transform	$x[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$X(z)$	R_x
time reversal	$x[-n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$X(\frac{1}{z})$	$\frac{1}{R_x}$
complex conjugation	$x^*[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$X^*(z^*)$	R_x
reversed conjugation	$x^*[-n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$X^*(\frac{1}{z^*})$	$\frac{1}{R_x}$
real part	$\mathfrak{Re}\{x[n]\}$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	R_x
imaginary part	$\mathfrak{Im}\{x[n]\}$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{1}{2j}[X(z)-X^*(z^*)]$	R_x
time shifting	$x[n-n_0]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$z^{-n_0}X(z)$	R_x
scaling in $\mathcal{Z}$	$a^n x[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$X\left(\frac{z}{a}\right)$	$\lvert a\rvert R_x$
downsampling by N	$x[Nn], N\in\mathbb{N}_0$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{1}{N}\sum_{k=0}^{N-1}X\left(W_N^k z^{\frac{1}{N}}\right) \quad W_N=e^{-\frac{j2\omega}{N}}$	R_x
linearity	$ax_1[n]+bx_2[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$aX_1(z)+bX_2(z)$	$R_x\cap R_y$
time multiplication	$x_1[n]x_2[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{1}{2\pi j}\oint X_1(u)X_2\left(\frac{z}{u}\right)u^{-1}du$	$R_x\cap R_y$
frequency convolution	$x_1[n]*x_2[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$X_1(z)X_2(t)$	$R_x\cap R_y$
delta function	$\delta[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	1	$\forall z$
shifted delta function	$\delta[n-n_0]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	z^{-n_0}	$\forall z$
step	$u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z}{z-1}$	$\lvert z\rvert>1$
	$-u[-n-1]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z}{z-1}$	$\lvert z\rvert<1$
ramp	$nu[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert>1$
	$n^2u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert>1$
	$-n^2u[-n-1]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert<1$
	$n^3u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$\lvert z\rvert>1$
	$-n^3u[-n-1]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$\lvert z\rvert<1$
	$(-1)^n$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z}{z+1}$	$\lvert z\rvert<1$
exponential	$a^nu[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z}{z-a}$	$\lvert z\rvert>\lvert a\rvert$
	$-a^nu[-n-1]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z}{z-a}$	$\lvert z\rvert<\lvert a\rvert$
	$a^{n-1}u[n-1]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{1}{z-a}$	$\lvert z\rvert>\lvert a\rvert$
	$na^nu[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert>\lvert a\rvert$
	$n^2a^nu[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{az(z+a}{(z-a)^3}$	$\lvert z\rvert>\lvert a\rvert$
	$e^{-an}u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z}{z-e^{-a}}$	$\lvert z\rvert>\lvert e^{-a}\rvert$
exp. interval	$\begin{cases}a^n & n=0,\ldots,N-1\\ 0 & \text{otherwise}\end{cases}$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{1-a^Nz^{-N}}{1-az^{-1}}$	$\lvert z\rvert>0$
sine	$\sin(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z\sin(\omega_0)}{z^2-2\cos(\omega_0)z+1}$	$\lvert z\rvert>1$
cosine	$\cos(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z(z-\cos(\omega_0))}{z^2-2\cos(\omega_0)z+1}$	$\lvert z\rvert>1$
	$a^n\sin(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{za\sin(\omega_0)}{z^2-2a\cos(\omega_0)z+a^2}$	$\lvert z\rvert>a$
	$a^n\cos(\omega_0 n)\,u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z(z-a\cos(\omega_0))}{z^2-2a\cos(\omega_0)z+a^2}$	$\lvert z\rvert>a$
differentiation in $\mathcal{Z}$	$nx[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$-z\frac{dX(z)}{dz}$	R_x
integration in $\mathcal{Z}$	$\frac{x[n]}{n}$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$-\int_0^z\frac{X(z)}{z}dz$	R_x
	$\frac{\prod_{i=1}^{m}(n-i+1)}{a^m m!}a^m u[n]$	$\overset{\mathcal{Z}}{\longleftrightarrow}$	$\frac{z}{(z-a)^{m+1}}$	

Note:

$$\frac{z}{z-1}=\frac{1}{1-z^{-1}}$$

Table of Common Discrete Time Fourier Transform (DTFT) Pairs

	$x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(e^{j\omega})e^{j\omega n}d\omega$		$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$	
transform	$x[n]$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega})$	
time reversal	$x[-n]$	$\xleftrightarrow{DTFT}$	$X(e^{-j\omega})$	
complex conjugation	$x^*[n]$	$\xleftrightarrow{DTFT}$	$X^*(e^{-j\omega})$	
reversed conjugation	$x^*[-n]$	$\xleftrightarrow{DTFT}$	$X^*(e^{j\omega})$	
	$x[n]$ is purely real	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = X^*(e^{-j\omega})$	even/symmetry
	$x[n]$ is purely imaginary	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$	odd/antisymmetry
even/symmetry	$x[n] = x^*[-n]$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega})$ is purely real	
odd/antisymmetry	$x[n] = -x^*[-n]$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega})$ is purely imaginary	
time shifting	$x[n-n_0]$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega})e^{-j\omega n_0}$	
	$x[n]e^{j\omega_0 n}$	$\xleftrightarrow{DTFT}$	$X(e^{j(\omega-\omega_0)})$	frequency shifting
downsampling by N	$x[Nn] \quad N \in \mathbb{N}_0$	$\xleftrightarrow{DTFT}$	$\frac{1}{N}\sum_{k=0}^{N-1} X(e^{j\frac{\omega-2\pi k}{N}})$	
upsampling by N	$\begin{cases} x\left[\frac{n}{N}\right] & n = kN \\ 0 & otherwise \end{cases}$	$\xleftrightarrow{DTFT}$	$X(e^{jN\omega})$	
linearity	$ax_1[n] + bx_2[n]$	$\xleftrightarrow{DTFT}$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$	
time multiplication	$x_1[n]x_2[n]$	$\xleftrightarrow{DTFT}$	$X_1(e^{j\omega}) * X_2(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X_1(e^{j(\omega-\sigma)})X_2(e^{j\sigma})d\sigma$	frequency convolution
frequency convolution	$x_1[n] * x_2[n]$	$\xleftrightarrow{DTFT}$	$X_1(e^{j\omega})X_2(e^{j\omega})$	frequency multiplication
delta function	$\delta[n]$	$\xleftrightarrow{DTFT}$	1	
shifted delta function	$\delta[n-n_0]$	$\xleftrightarrow{DTFT}$	$e^{-j\omega n_0}$	
	1	$\xleftrightarrow{DTFT}$	$\tilde{\delta}(\omega)$	delta function
	$e^{j\omega_0 n}$	$\xleftrightarrow{DTFT}$	$\tilde{\delta}(\omega-\omega_0)$	shifted delta function
sine	$\sin(\omega_0 n + \phi)$	$\xleftrightarrow{DTFT}$	$\frac{j}{2}[e^{-j\phi}\tilde{\delta}(\omega+\omega_0+2\pi k) - e^{+j\phi}\tilde{\delta}(\omega-\omega_0+2\pi k)]$	
cosine	$\cos(\omega_0 n + \phi)$	$\xleftrightarrow{DTFT}$	$\frac{1}{2}[e^{-j\phi}\tilde{\delta}(\omega+\omega_0+2\pi k) + e^{+j\phi}\tilde{\delta}(\omega-\omega_0+2\pi k)]$	
rectangular	$\text{rect}\left(\frac{n}{M}\right) = \begin{cases} 1 & \lvert n\rvert \leqslant M \\ 0 & \text{otherwise} \end{cases}$	$\xleftrightarrow{DTFT}$	$\frac{\sin[\omega(M+\frac{1}{2})]}{\sin(\omega/2)}$	
step	$u[n]$	$\xleftrightarrow{DTFT}$	$\frac{1}{1-e^{-j\omega}} + \frac{1}{2}\tilde{\delta}(\omega)$	
decaying step	$a^n u[n] \quad (\lvert a\rvert < 1)$	$\xleftrightarrow{DTFT}$	$\frac{1}{1-ae^{-j\omega}}$	
special decaying step	$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\xleftrightarrow{DTFT}$	$\frac{1}{(1-ae^{-j\omega})^2}$	
sinc	$\frac{\sin(\omega_c n)}{\pi n} = \frac{\omega_c}{\pi}\,\text{sinc}(\omega_c n)$	$\xleftrightarrow{DTFT}$	$\widetilde{\text{rect}}\left(\frac{\omega}{\omega_c}\right) = \begin{cases} 1 & \lvert\omega\rvert < \omega_c \\ 0 & \omega_c < \lvert\omega\rvert < \pi \end{cases}$	
MA	$\text{rect}\left(\frac{n}{M} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leqslant n \leqslant M \\ 0 & \text{otherwise} \end{cases}$	$\xleftrightarrow{DTFT}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}e^{-j\omega M/2}$	
MA	$\text{rect}\left(\frac{n}{M-1} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leqslant n \leqslant M-1 \\ 0 & \text{otherwise} \end{cases}$	$\xleftrightarrow{DTFT}$	$\frac{\sin[\omega M/2]}{\sin(\omega/2)}e^{-j\omega(M-1)/2}$	
derivation	$nx[n]$	$\xleftrightarrow{DTFT}$	$j\frac{d}{d\omega}X(e^{j\omega})$	
difference	$x[n] - x[n-1]$	$\xleftrightarrow{DTFT}$	$(1-e^{-j\omega})X(e^{j\omega})$	
	$\frac{a^n \sin[\omega_0(n+1)]}{\sin\omega_0}u[n] \quad \lvert a\rvert < 1$	$\xleftrightarrow{DTFT}$	$\frac{1}{1-2a\cos(\omega_0 e^{-j\omega})+a^2e^{-j2\omega}}$	

Note:

$$\tilde{\delta}(\omega) = \sum_{k=-\infty}^{+\infty} \delta(\omega + 2\pi k) \qquad \widetilde{\text{rect}}(\omega) = \sum_{k=-\infty}^{+\infty} \text{rect}(\omega + 2\pi k)$$

Parseval:

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} |X(e^{j\omega})|^2 d\omega$$

Table of Laplace Transform Pairs

	$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j}\lim_{T\to\infty}\int_{c-jT}^{c+jT} F(s)e^{st}ds$		$F(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$	
transform	$f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F(s)$	
complex conjugation	$f^*(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F^*(s^*)$	
time shifting	$f(t-a) \quad t \geqslant a > 0$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$a^{-as}F(s)$	
	$e^{-at}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F(s+a)$	frequency shifting
time scaling	$f(at)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{\lvert a\rvert}F(\frac{s}{a})$	
linearity	$af_1(t) + bf_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$aF_1(s) + bF_2(s)$	
time multiplication	$f_1(t)f_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F_1(s) * F_2(s)$	frequency convolution
time convolution	$f_1(t) * f_2(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$F_1(s)F_2(s)$	frequency product
delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1	
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}	exponential decay
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$	
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$	
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$	
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$	
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$	
two-sided exponential decay	$e^{-a\lvert t\rvert}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$	
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$	
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$	
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$	
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$	
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$	
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$	
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$	
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$	
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$	
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$	
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$	
time differentiation	$f'(t) = \frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s) - f(0)$	
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s) - sf(0) - f'(0)$	
time n-th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$	
time integration	$\int_0^t f(\tau)d\tau = (u * f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$	
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$	
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$	
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \ldots + \frac{f^{-n}(0)}{s}$	