APPENDIX D

Laplace Transform Pairs

Table D.1

$F(s)$	**$f(t),\ t \supseteq 0$**
1. 1	$\delta(t_0)$, unit impulse at $t = t_0$
2. $1/s$	1, unit step
3. $\dfrac{n!}{s^{n+1}}$	t^n
4. $\dfrac{1}{(s+a)}$	e^{-at}
5. $\dfrac{1}{(s+a)^n}$	$\dfrac{1}{(n-1)!}\,t^{n-1}e^{-at}$
6. $\dfrac{a}{s(s+a)}$	$1 - e^{-at}$
7. $\dfrac{1}{(s+a)(s+b)}$	$\dfrac{1}{(b-a)}\,(e^{-at} - e^{-bt})$
8. $\dfrac{s+\alpha}{(s+a)(s+b)}$	$\dfrac{1}{(b-a)}\,[(\alpha - a)e^{-at} - (\alpha - b)e^{-bt}]$
9. $\dfrac{ab}{s(s+a)(s+b)}$	$1 - \dfrac{b}{(b-a)}e^{-at} + \dfrac{a}{(b-a)}\,e^{-bt}$
10. $\dfrac{1}{(s+a)(s+b)(s+c)}$	$\dfrac{e^{-at}}{(b-a)(c-a)} + \dfrac{e^{-bt}}{(c-a)(a-b)} + \dfrac{e^{-ct}}{(a-c)(b-c)}$
11. $\dfrac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\dfrac{(\alpha-a)e^{-at}}{(b-a)(c-a)} + \dfrac{(\alpha-b)e^{-bt}}{(c-b)(a-b)} + \dfrac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
12. $\dfrac{ab(s+\alpha)}{s(s+a)(s+b)}$	$\alpha - \dfrac{b(\alpha-a)}{(b-a)}\,e^{-at} + \dfrac{a(\alpha-b)}{(b-a)}e^{-bt}$
13. $\dfrac{\omega}{s^2+\omega^2}$	$\sin \omega t$
14. $\dfrac{s}{s^2+\omega^2}$	$\cos \omega t0$

Table D.1 continued

Table D.1 *Continued*

$F(s)$	$f(t),\ t \supseteq 0$
15. $\dfrac{s+\alpha}{s^2+\omega^2}$	$\dfrac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t+\phi),\ \phi=\tan^{-1}\omega/\alpha$
16. $\dfrac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
17. $\dfrac{(s+\alpha)}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
18. $\dfrac{s+\alpha}{(s+a)^2+\omega^2}$	$\dfrac{1}{\omega}[(\alpha-a)^2+\omega^2]^{1/2}e^{-at}\sin(\omega t+\phi),$ $\phi=\tan^{-1}\dfrac{\omega}{\alpha-a}$
19. $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t,\ \zeta<1$
20. $\dfrac{1}{s[(s+a)^2+\omega^2]}$	$\dfrac{1}{a^2+\omega^2}+\dfrac{1}{\omega\sqrt{a^2+\omega^2}}e^{-at}\sin(\omega t-\phi),$ $\phi=\tan^{-1}\dfrac{\omega}{-a}$
21. $\dfrac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi),$ $\phi=\cos^{-1}\zeta,\ \zeta<1$
22. $\dfrac{(s+\alpha)}{s[(s+a)^2+\omega^2]}$	$\dfrac{\alpha}{a^2+\omega^2}+\dfrac{1}{\omega}\left[\dfrac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2}e^{-at}\sin(\omega t+\phi),$ $\phi=\tan^{-1}\dfrac{\omega}{\alpha-a}-\tan^{-1}\dfrac{\omega}{-a}$
23. $\dfrac{1}{(s+c)[(s+a)^2+\omega^2]}$	$\dfrac{e^{-ct}}{(c-a)^2+\omega^2}+\dfrac{e^{-at}\sin(\omega t+\phi)}{\omega[(c-a)^2+\omega^2]^{1/2}},\ \phi=\tan^{-1}\dfrac{\omega}{c-a}$